CLEP®

Test Information Guide: College-Level Examination Program®

2015-16

Calculus

CollegeBoard

CLEP TEST INFORMATION GUIDE FOR CALCULUS

History of CLEP

Since 1967, the College-Level Examination Program (CLEP®) has provided over six million people with the opportunity to reach their educational goals. CLEP participants have received college credit for knowledge and expertise they have gained through prior course work, independent study or work and life experience.

Over the years, the CLEP examinations have evolved to keep pace with changing curricula and pedagogy. Typically, the examinations represent material taught in introductory college-level courses from all areas of the college curriculum. Students may choose from 33 different subject areas in which to demonstrate their mastery of college-level material.

Today, more than 2,900 colleges and universities recognize and grant credit for CLEP.

Philosophy of CLEP

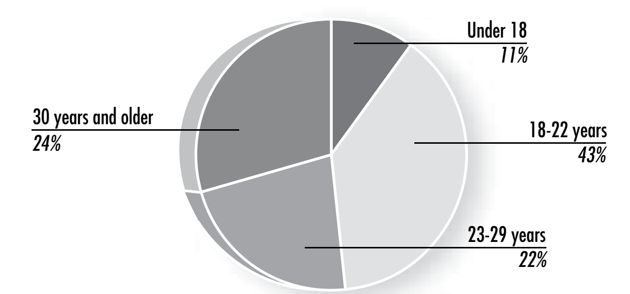
Promoting access to higher education is CLEP's foundation. CLEP offers students an opportunity to demonstrate and receive validation of their college-level skills and knowledge. Students who achieve an appropriate score on a CLEP exam can enrich their college experience with higher-level courses in their major field of study, expand their horizons by taking a wider array of electives and avoid repetition of material that they already know.

CLEP Participants

CLEP's test-taking population includes people of all ages and walks of life. Traditional 18- to 22-year-old students, adults just entering or returning to school, high-school students, home-schoolers and international students who need to quantify their knowledge have all been assisted by CLEP in earning their college degrees. Currently, 59 percent of CLEP's National (civilian) test-takers are women and 46 percent are 23 years of age or older.

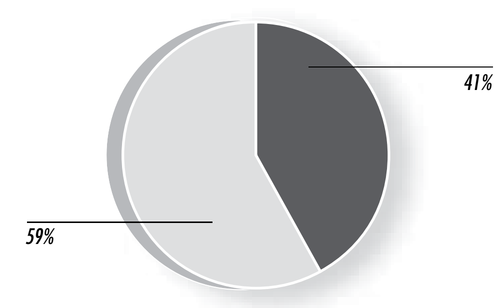
For over 30 years, the College Board has worked to provide government-funded credit-by-exam opportunities to the military through CLEP. Military service members are fully funded for their CLEP exam fees. Exams are administered at military installations worldwide through computer-based testing programs. Approximately one-third of all CLEP candidates are military service members.

2014-15 National CLEP Candidates by Age*



* These data are based on 100% of CLEP test-takers who responded to this survey question during their examinations.

2014-15 National CLEP Candidates by Gender



Computer-Based CLEP Testing

The computer-based format of CLEP exams allows for a number of key features. These include:

- a variety of question formats that ensure effective assessment
- real-time score reporting that gives students and colleges the ability to make immediate credit-granting decisions (except College Composition, which requires faculty scoring of essays twice a month)
- a uniform recommended credit-granting score of 50 for all exams
- "rights-only" scoring, which awards one point per correct answer
- pretest questions that are not scored but provide current candidate population data and allow for rapid expansion of question pools

CLEP Exam Development

Content development for each of the CLEP exams is directed by a test development committee. Each committee is composed of faculty from a wide variety of institutions who are currently teaching the relevant college undergraduate courses. The committee members establish the test specifications based on feedback from a national curriculum survey; recommend credit-granting scores and standards; develop and select test questions; review statistical data and prepare descriptive material for use by faculty (*Test Information Guides*) and students planning to take the tests (*CLEP Official Study Guide*).

College faculty also participate in CLEP in other ways: they convene periodically as part of standard-setting panels to determine the recommended level of student competency for the granting of college credit; they are called upon to write exam questions and to review exam forms; and they help to ensure the continuing relevance of the CLEP examinations through the curriculum surveys.

The Curriculum Survey

The first step in the construction of a CLEP exam is a curriculum survey. Its main purpose is to obtain information needed to develop test-content specifications that reflect the current college curriculum and to recognize anticipated changes in the field. The surveys of college faculty are conducted in each subject every few years depending on the discipline. Specifically, the survey gathers information on:

- the major content and skill areas covered in the equivalent course and the proportion of the course devoted to each area
- specific topics taught and the emphasis given to each topic
- specific skills students are expected to acquire and the relative emphasis given to them
- recent and anticipated changes in course content, skills and topics
- the primary textbooks and supplementary learning resources used
- titles and lengths of college courses that correspond to the CLEP exam

The Committee

The College Board appoints standing committees of college faculty for each test title in the CLEP battery. Committee members usually serve a term of up to four years. Each committee works with content specialists at Educational Testing Service to establish test specifications and develop the tests. Listed below are the current committee members and their institutional affiliations.

Chaim Goodman-Strauss, *Chair*	University of Arkansas, Fayetteville
Kodwo Annan	Georgia Gwinnett College
Adrienne Stanley	University of Northern Iowa

The primary objective of the committee is to produce tests with good content validity. CLEP tests must be rigorous and relevant to the discipline and the appropriate courses. While the consensus of the committee members is that this test has high content validity for a typical introductory Calculus course or curriculum, the validity of the content for a specific course or curriculum is best determined locally through careful review and comparison of test content, with instructional content covered in a particular course or curriculum.

The Committee Meeting

The exam is developed from a pool of questions written by committee members and outside question writers. All questions that will be scored on a CLEP exam have been pretested; those that pass a rigorous statistical analysis for content relevance, difficulty, fairness and correlation with assessment criteria are added to the pool. These questions are compiled by test development specialists according to the test specifications, and are presented to all the committee members for a final review. Before convening at a two- or three-day committee meeting, the members have a chance to review the test specifications and the pool of questions available for possible inclusion in the exam.

At the meeting, the committee determines whether the questions are appropriate for the test and, if not, whether they need to be reworked and pretested again to ensure that they are accurate and unambiguous. Finally, draft forms of the exam are reviewed to ensure comparable levels of difficulty and content specifications on the various test forms. The committee is also responsible for writing and developing pretest questions. These questions are administered to candidates who take the examination and provide valuable statistical feedback on student performance under operational conditions.

Once the questions are developed and pretested, tests are assembled in one of two ways. In some cases, test forms are assembled in their entirety. These forms are of comparable difficulty and are therefore interchangeable. More commonly, questions are assembled into smaller, content-specific units called testlets, which can then be combined in different ways to create multiple test forms. This method allows many different forms to be assembled from a pool of questions.

Test Specifications

Test content specifications are determined primarily through the curriculum survey, the expertise of the committee and test development specialists, the recommendations of appropriate councils and conferences, textbook reviews and other appropriate sources of information. Content specifications take into account:

- the purpose of the test
- the intended test-taker population
- the titles and descriptions of courses the test is designed to reflect
- the specific subject matter and abilities to be tested
- the length of the test, types of questions and instructions to be used

Recommendation of the American Council on Education (ACE)

The American Council on Education's College Credit Recommendation Service (ACE CREDIT) has evaluated CLEP processes and procedures for developing, administering and scoring the exams. Effective July 2001, ACE recommended a uniform credit-granting score of 50 across all subjects (with additional Level-2 recommendations for the world language examinations), representing the performance of students who earn a grade of C in the corresponding course. Every test title has a minimum score of **20**, a maximum score of **80** and a cut score of **50**. However, these score values cannot be compared across exams. The score scale is set so that a score of **50** represents the performance expected of a typical C student, which may differ from one subject to another. The score scale is not based on actual performance of test-takers. It is derived from the judgment of a panel of experts (college faculty who teach an equivalent course) who provide information on the level of student performance that would be necessary to receive college credit in the course.

Over the years, the CLEP examinations have been adapted to adjust to changes in curricula and pedagogy. As academic disciplines evolve, college faculty incorporate new methods and theory into their courses. CLEP examinations are revised to reflect those changes so the examinations continue to meet the needs of colleges and students. The CLEP program's most recent ACE CREDIT review was held in June 2015.

The American Council on Education, the major coordinating body for all the nation's higher education institutions, seeks to provide leadership and a unifying voice on key higher education issues and to influence public policy through advocacy, research and program initiatives. For more information, visit the ACE CREDIT website at www.acenet.edu/acecredit.

CLEP Credit Granting

CLEP uses a common recommended credit-granting score of 50 for all CLEP exams.

This common credit-granting score does not mean, however, that the standards for all CLEP exams are the same. When a new or revised version of a test is introduced, the program conducts a standard setting to determine the recommended credit-granting score ("cut score").

A standard-setting panel, consisting of 15–20 faculty members from colleges and universities across the country who are currently teaching the course, is appointed to give its expert judgment on the level of student performance that would be necessary to receive college credit in the course. The panel reviews the test and test specifications and defines the capabilities of the typical A student, as well as those of the typical B, C and D students.* Expected individual student performance is rated by each panelist on each question. The combined average of the ratings is used to determine a recommended number of examination questions that must be answered correctly to mirror classroom performance of typical B and C students in the related course. The panel's findings are given to members of the test development committee who, with the help of Educational Testing Service and College Board psychometric specialists, make a final determination on which raw scores are equivalent to B and C levels of performance.

*Student performance for the language exams (French, German and Spanish) is defined only at the B and C levels.

Calculus

Description of the Examination

The Calculus examination covers skills and concepts that are usually taught in a one-semester college course in calculus. The content of each examination is approximately 60% limits and differential calculus and 40% integral calculus. Algebraic, trigonometric, exponential, logarithmic and general functions are included. The exam is primarily concerned with an intuitive understanding of calculus and experience with its methods and applications. Knowledge of preparatory mathematics, including algebra, geometry, trigonometry and analytic geometry is assumed.

The examination contains 44 questions, in two sections, to be answered in approximately 90 minutes. Any time candidates spend on tutorials and providing personal information is in addition to the actual testing time.

- Section 1: 27 questions, approximately 50 minutes. No calculator is allowed for this section.
- Section 2: 17 questions, approximately 40 minutes. The use of an **online graphing calculator (non-CAS)** is allowed for this section. Only some of the questions will require the use of the calculator.

Graphing Calculator

A graphing calculator is integrated into the exam software, and it is available to students during Section 2 of the exam. Since only some of the questions in Section 2 actually require the calculator, students are expected to know how and when to make appropriate use of it. The graphing calculator, together with a brief tutorial, is available to students as a free download for a 90-day trial period. **Students are expected to download the calculator and become familiar with its functionality prior to taking the exam.**

For more information about downloading the practice version of the graphing calculator, please visit the Calculus exam description on the CLEP website, clep.collegeboard.org.

In order to answer some of the questions in Section 2 of the exam, students may be required to use the online graphing calculator in the following ways:

- Perform calculations (e.g., exponents, roots, trigonometric values, logarithms).
- Graph functions and analyze the graphs.
- Find zeros of functions.
- Find points of intersection of graphs of functions.
- Find minima/maxima of functions.
- Find numerical solutions to equations.
- Generate a table of values for a function.

Knowledge and Skills Required

Questions on the exam require candidates to demonstrate the following abilities:

- Solving routine problems involving the techniques of calculus (approximately 50% of the exam)
- Solving nonroutine problems involving an understanding of the concepts and applications of calculus (approximately 50% of the exam)

The subject matter of the Calculus exam is drawn from the following topics. The percentages next to the main topics indicate the approximate percentage of exam questions on that topic.

10% **Limits**

- Statement of properties, e.g., limit of a constant, sum, product or quotient
- Limit calculations, including limits involving infinity, e.g., $\lim_{x \to 0} \frac{\sin x}{x} = 1$, $\lim_{x \to 0} \frac{1}{x}$ is nonexistent and $\lim_{x \to \infty} \frac{\sin x}{x} = 0$
- Continuity

50% **Differential Calculus**

The Derivative

- Definitions of the derivative e.g., $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ and $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
- Derivatives of elementary functions
- Derivatives of sums, products and quotients (including $\tan x$ and $\cot x$)
- Derivative of a composite function (chain rule), e.g., $\sin(ax + b)$, ae^{kx}, $\ln(kx)$
- Implicit differentiation
- Derivative of the inverse of a function (including $\arcsin x$ and $\arctan x$)
- Higher order derivatives
- Corresponding characteristics of graphs of f, f', and f''
- Statement of the Mean Value Theorem; applications and graphical illustrations
- Relation between differentiability and continuity
- Use of L'Hospital's Rule (quotient and indeterminate forms)

Applications of the Derivative

- Slope of a curve at a point
- Tangent lines and linear approximation
- Curve sketching: increasing and decreasing functions; relative and absolute maximum and minimum points; concavity; points of inflection
- Extreme value problems
- Velocity and acceleration of a particle moving along a line
- Average and instantaneous rates of change
- Related rates of change

40% **Integral Calculus**

Antiderivatives and Techniques of Integration

- Concept of antiderivatives
- Basic integration formulas
- Integration by substitution (use of identities, change of variable)

Applications of Antiderivatives

- Distance and velocity from acceleration with initial conditions
- Solutions of $y' = ky$ and applications to growth and decay

The Definite Integral

- Definition of the definite integral as the limit of a sequence of Riemann sums and approximations of the definite integral using areas of rectangles
- Properties of the definite integral
- The Fundamental Theorem: $\frac{d}{dx} \int_a^x f(t)\,dt = f(x)$ and $\int_a^b F'(x)\,dx = F(b) - F(a)$

Applications of the Definite Integral

- Average value of a function on an interval
- Area, including area between curves
- Other (e.g., accumulated change from a rate of change)

Notes and Reference Information

(1) Figures that accompany questions are intended to provide information useful in answering the questions. All figures lie in a plane unless otherwise indicated. The figures are drawn as accurately as possible EXCEPT when it is stated in a specific question that the figure is not drawn to scale. Straight lines and smooth curves may appear slightly jagged.

(2) Unless otherwise specified, all angles are measured in radians, and all numbers used are real numbers.

(3) Unless otherwise specified, the domain of any function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. The range of f is assumed to be the set of all real numbers $f(x)$ where x is in the domain of f.

(4) In this test, $\ln x$ denotes the natural logarithm of x (that is, the logarithm to the base e).

(5) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

Sample Test Questions

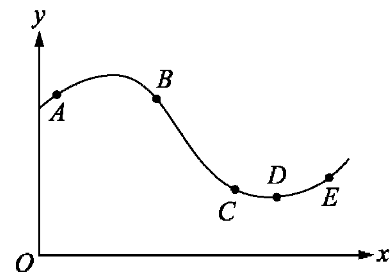
The following sample questions do not appear on an actual CLEP Examination. They are intended to give potential test-takers an indication of the format and difficulty level of the examination, and to provide content for practice and review. Knowing the correct answers to all of the sample questions is not a guarantee of satisfactory performance on the exam.

Section I

Directions: A calculator will not be available for questions in this section. Some questions will require you to select from among five choices. For these questions, select the BEST of the choices given. Some questions will require you to enter a numerical answer in the box provided.

1. If $y = (x+1)(x-1)(x+5)$, then $\frac{dy}{dx} =$

(A) $x^2 - 1$
(B) $2x^2 + 10x$
(C) $3x^2 + 10x - 1$
(D) $x^3 + 5x^2 - x$
(E) $2x^3 + 20x^2 + 50x$



2. At which of the five points on the graph in the figure above are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

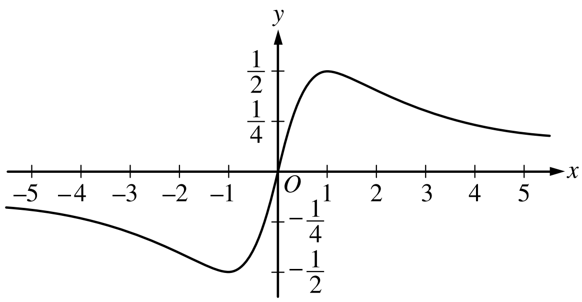
(A) A (B) B (C) C (D) D (E) E

3. Which of the following is an equation of the line tangent to the graph of $f(x) = x^3 - x$ at the point where $x = 2$?

(A) $y - 6 = 4(x-2)$
(B) $y - 6 = 5(x-2)$
(C) $y - 6 = 6(x-2)$
(D) $y - 6 = 11(x-2)$
(E) $y - 6 = 12(x-2)$

4. $\int (e^x + e)\, dx =$

(A) $e^x + C$
(B) $e^x + e + C$
(C) $e^x + ex + C$
(D) $\frac{e^{x+1}}{x+1} + ex + C$
(E) $\frac{e^{x+1}}{x+1} + \frac{e^2}{2} + C$



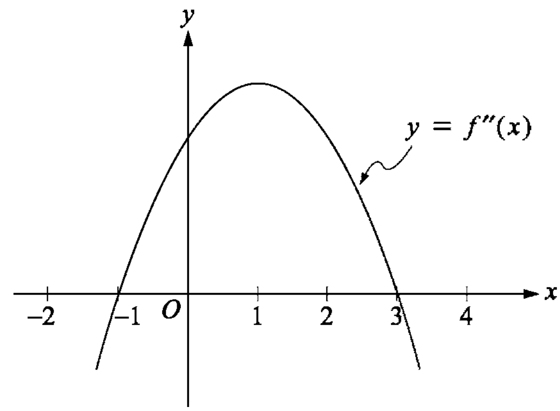
5. The graph of the function $f(x) = \dfrac{x}{1+x^2}$ is shown in the figure above. Which of the following statements is true?

I. $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to \infty} f(x)$

II. $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x)$

III. $\lim\limits_{x \to 1} \dfrac{f(x) - \frac{1}{2}}{x - 1}$ does not exist.

(A) None
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III



6. The graph of f'', the second derivative of the function f, is shown in the figure above. On what intervals is the graph of f concave up?

(A) $(-\infty, \infty)$
(B) $(-\infty, -1)$ and $(3, \infty)$
(C) $(-\infty, 1)$
(D) $(-1, 3)$
(E) $(1, \infty)$

7. $\int (x-1)\sqrt{x}\, dx =$

(A) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

(B) $\frac{1}{2}x^2 + 2x^{\frac{2}{3}} - x + C$

(C) $\frac{1}{2}x^2 - x + C$

(D) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

(E) $\frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + C$

8. Let f and g be the functions defined by $f(x) = \sin x$ and $g(x) = \cos x$. For which of the following values of a is the line tangent to the graph of f at $x = a$ parallel to the line tangent to the graph of g at $x = a$?

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{4}$ (E) π

9. The acceleration, at time t, of a particle moving along the x-axis is given by $a(t) = 20t^3 + 6$. At time $t = 0$, the velocity of the particle is 0 and the position of the particle is 7. What is the position of the particle at time t?

(A) $120t + 7$

(B) $60t^2 + 7t$

(C) $5t^4 + 6t + 7$

(D) $t^5 + 3t^2 + 7$

(E) $t^5 + 3t^2 + 7t$

10. If $f(x) = \frac{\sin x}{2x}$, then $f'(x) =$

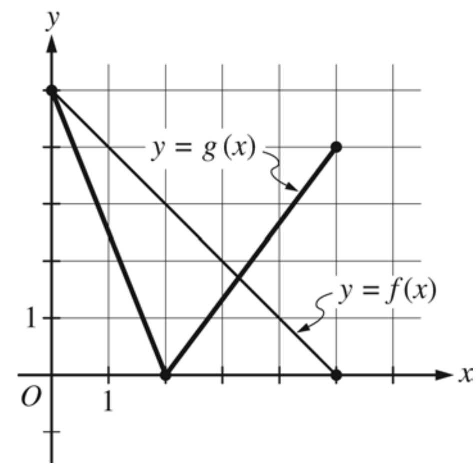
(A) $\frac{\cos x}{2}$

(B) $\frac{x\cos x - \sin x}{2x^2}$

(C) $\frac{x\cos x - \sin x}{4x^2}$

(D) $\frac{\sin x - x\cos x}{2x^2}$

(E) $\frac{\sin x - x\cos x}{4x^2}$



11. The piecewise linear graphs of the functions f and g are shown in the figure above. If $h(x) = f(g(x))$, what is the value of $h'(3)$?

(A) -4

(B) $-\frac{4}{3}$

(C) -1

(D) 1

(E) $\frac{4}{3}$

12. What is $\lim_{h\to 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\frac{\pi}{2}}{h}$?

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

13. If $x^2 + y^3 = x^3y^2$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{2x + 3y^2 - 3x^2y^2}{2x^3y}$

(B) $\dfrac{2x^3y + 3x^2y^2 - 2x}{3y^2}$

(C) $\dfrac{3x^2y^2 - 2x}{3y^2 - 2x^3y}$

(D) $\dfrac{3y^2 - 2x^3y}{3x^2y^2 - 2x}$

(E) $\dfrac{6x^2y - 2x}{3y^2}$

14. For which of the following functions does $\dfrac{d^3y}{dx^3} = \dfrac{dy}{dx}$?

I. $y = e^x$
II. $y = e^{-x}$
III. $y = \sin x$

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

15. What is $\lim\limits_{x\to 0^-} \dfrac{2x}{|x|}$?

(A) -2
(B) -1
(C) 0
(D) 2
(E) The limit does not exist.

16. Which of the following statements about the curve $y = x^4 - 2x^3$ is true?

(A) The curve has no relative extremum.
(B) The curve has one point of inflection and two relative extrema.
(C) The curve has two points of inflection and one relative extremum.
(D) The curve has two points of inflection and two relative extrema.
(E) The curve has two points of inflection and three relative extrema.

17. $\dfrac{d}{dx}\left(\dfrac{\sin x + 1}{\cos x}\right) =$

(A) $-\csc^2 x - \sec x \tan x$

(B) $-\csc^2 x + \sec x \tan x$

(C) 0

(D) $\sec^2 x + \sec x \tan x$

(E) $\sec^2 x + \tan^2 x$

18. Let f be the function defined by

$$f(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{for } x \neq 5 \\ 0 & \text{for } x = 5 \end{cases}.$$

Which of the following statements about f are true?

I. $\lim\limits_{x\to 5} f(x)$ exists.
II. $f(5)$ exists.
III. $f(x)$ is continuous at $x = 5$.

(A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

19. What is the average rate of change of the function f defined by $f(x) = 100 \cdot 2^x$ on the interval $[0, 4]$?

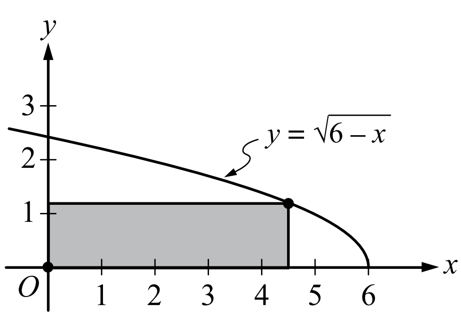
(A) 100
(B) 375
(C) 400
(D) 1,500
(E) 1,600

20. If the functions f and g are defined for all real numbers and f is an antiderivative of g, which of the following statements is NOT necessarily true?

(A) If $g(x) > 0$ for all x, then f is increasing.
(B) If $g(a) = 0$, then the graph of f has a horizontal tangent at $x = a$.
(C) If $f(x) = 0$ for all x, then $g(x) = 0$ for all x.
(D) If $g(x) = 0$ for all x, then $f(x) = 0$ for all x.
(E) f is continuous for all x.

21. If $f(x) = \arctan(\pi x)$, then $f'(0) =$

(A) $-\pi$ (B) -1 (C) 0 (D) 1 (E) π



22. Consider a rectangle in the xy-plane with its lower-left vertex at the origin and its upper-right vertex on the graph of $y = \sqrt{6 - x}$, as indicated in the figure above. What is the maximum area of such a rectangle?

(A) $\sqrt{6}$
(B) 4
(C) $3\sqrt{3}$
(D) $4\sqrt{2}$
(E) $4\sqrt{6}$

23. Let $f(x) = x^3 + x$. If h is the inverse function of f, then $h'(2) =$

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) 1 (D) 4 (E) 13

24. Let F be the number of trees in a forest at time t, in years. If F is decreasing at a rate given by the equation $\frac{dF}{dt} = -2F$ and if $F(0) = 5000$, then $F(t) =$

(A) $5000t^{-2}$
(B) $5000e^{-2t}$
(C) $5000 - 2t$
(D) $5000 + t^{-2}$
(E) $5000 + e^{-2t}$

25. Let f be the function defined by $f(x) = \sqrt{x}$. Using the line tangent to the graph of f at $x = 9$, what is the approximation of $f(9.3) = \sqrt{9.3}$?

26. What is the area of the region in the first quadrant that is bounded by the line $y = 6x$ and the parabola $y = 3x^2$?

27. Let f be a differentiable function defined on the closed interval $[a, b]$ and let c be a point in the open interval (a, b) such that

- $f'(c) = 0$,
- $f'(x) > 0$ when $a \le x < c$, and
- $f'(x) < 0$ when $c < x \le b$.

Which of the following statements must be true?

(A) $f(c) = 0$

(B) $f''(c) = 0$

(C) $f(c)$ is an absolute maximum value of f on $[a, b]$.

(D) $f(c)$ is an absolute minimum value of f on $[a, b]$.

(E) The graph of f has a point of inflection at $x = c$.

28. The function f is continuous on the open interval $(-\pi, \pi)$. If $f(x) = \dfrac{\cos x - 1}{x \sin x}$ for $x \ne 0$, what is the value of $f(0)$?

(A) -1 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 1

$$g(20) = 0$$
$$g'(t) > 0 \text{ for all values of } t$$

29. The function g is differentiable and satisfies the conditions above. Let F be the function given by $F(x) = \int_0^x g(t)\,dt$. Which of the following must be true?

(A) F has a local minimum at $x = 20$.

(B) F has a local maximum at $x = 20$.

(C) The graph of F has a point of inflection at $x = 20$.

(D) F has no local minima or local maxima on the interval $0 \le x < \infty$.

(E) $F'(20)$ does not exist.

30. The Riemann sum $\displaystyle\sum_{i=1}^{50} \left(\frac{i}{50}\right)^2 \frac{1}{50}$ on the closed interval $[0, 1]$ is an approximation for which of the following definite integrals?

(A) $\int_0^1 x^2\,dx$

(B) $\int_0^{50} x^2\,dx$

(C) $\int_0^1 \left(\frac{x}{50}\right)^2 dx$

(D) $\int_0^{50} \left(\frac{x}{50}\right)^2 dx$

(E) $\int_0^1 \frac{x^2}{50^3}\,dx$

31. $\int_0^6 \left|x^2 - 6x + 8\right| dx =$

(A) $\frac{4}{3}$

(B) $\frac{20}{3}$

(C) $\frac{44}{3}$

(D) 12

(E) 228

32. A particle moves along the x-axis, and its velocity at time t is given by $v(t) = t^3 - 3t^2 + 12t + 8$. What is the maximum acceleration of the particle on the interval $0 \le t \le 3$?

(A) 9

(B) 12

(C) 14

(D) 21

(E) 44

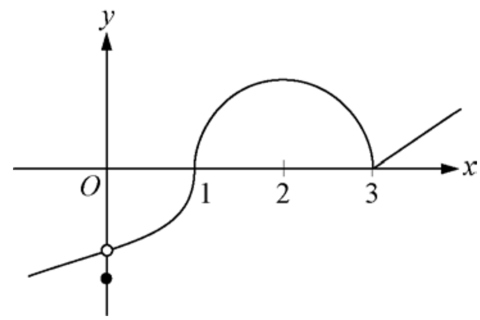
$$f(x) = \begin{cases} \dfrac{x}{2-\sqrt{4+x}} & \text{for } x \neq 0 \text{ and } x \geq -4 \\ k & \text{for } x = 0 \end{cases}$$

33. Let f be the function defined above, where k is a constant. For what value of k is f continuous at $x = 0$?

$k =$ ☐

34. If f is a continuous, even function such that $\int_0^3 f(x)\,dx = -4$, then $\int_{-3}^{3} (5f(x)+1)\,dx =$

(A) -39
(B) -34
(C) -19
(D) -14
(E) 6



35. The graph of the function f, shown in the figure above, has a vertical tangent at $x = 1$ and a horizontal tangent at $x = 2$.

For each of the following values of x, indicate whether f is not continuous, continuous but not differentiable, or differentiable.

x	f is not continuous at x.	f is continuous but not differentiable at x.	f is differentiable at x.
0			
1			
2			
3			

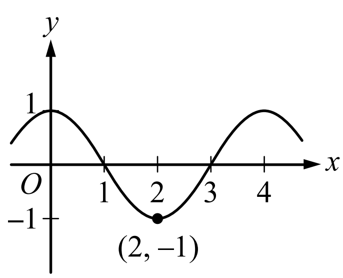
Click on your choices.

Section II

Directions: A graphing calculator will be available for the questions in this section. Some questions will require you to select from among five choices. For these questions, select the BEST of the choices given. If the exact numerical value of your answer is not one of the choices, select the choice that best approximates this value. Some questions will require you to enter a numerical answer in the box provided.

36. $\int_{5}^{10} \frac{\ln(10x)}{x}\, dx =$

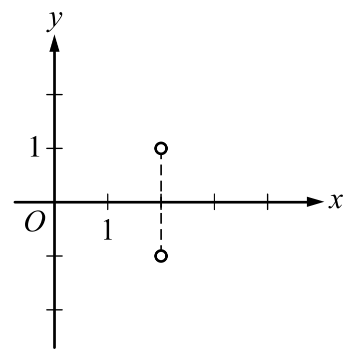
(A) 1.282
(B) 2.952
(C) 5.904
(D) 6.797
(E) 37.500



Graph of f

37. The graph of a function f is given above. What is $\lim_{x \to 2} \frac{x}{f(x)+1}$?

(A) $-\infty$
(B) -1
(C) $\frac{2}{3}$
(D) 1
(E) ∞



Graph of f

38. Let F be a continuous function. The graph of f, the derivative of F, is given above. Which of the following statements is false?

(A) $F(2) - F(0) > 0$
(B) F is defined at $x = 2$.
(C) F is differentiable at $x = 2$.
(D) F has a relative maximum at $x = 2$.
(E) F is decreasing on $[2, \infty)$.

39. Let f be a function with second derivative given by $f''(x) = \sin(2x) - \cos(4x)$. How many points of inflection does the graph of f have on the interval $[0, 10]$?

(A) Six
(B) Seven
(C) Eight
(D) Ten
(E) Thirteen

40. The area of the region in the first quadrant between the graph of $y = x\sqrt{4 - x^2}$ and the x-axis is

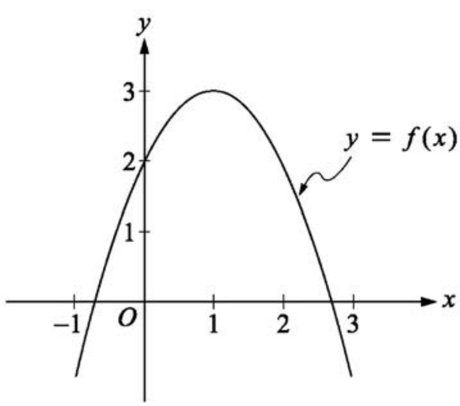
(A) $\frac{2}{3}\sqrt{2}$
(B) $\frac{8}{3}$
(C) $2\sqrt{2}$
(D) $2\sqrt{3}$
(E) $\frac{16}{3}$

41. The function f is given by $f(x) = 3x^2 + 1$. What is the average value of f over the closed interval $[1, 3]$?

$\boxed{}$

42. Starting at $t = 0$, a particle moves along the x-axis so that its position at time t is given by $x(t) = t^4 - 5t^2 + 2t$. What are all values of t for which the particle is moving to the left?

(A) $0 < t < 0.913$
(B) $0.203 < t < 1.470$
(C) $0.414 < t < 0.913$
(D) $0.414 < t < 2.000$
(E) There are no values of t for which the particle is moving to the left.



43. The function f has a relative maximum value of 3 at $x = 1$, as shown in the figure above. If $h(x) = x^2 f(x)$, then $h'(1) =$

(A) -6 (B) -3 (C) 0 (D) 3 (E) 6

44. $\int \cos^2 x \sin x \, dx =$

(A) $-\dfrac{\cos^3 x}{3} + C$

(B) $-\dfrac{\cos^3 x \sin^2 x}{6} + C$

(C) $\dfrac{\sin^2 x}{2} + C$

(D) $\dfrac{\cos^3 x}{3} + C$

(E) $\dfrac{\cos^3 x \sin^2 x}{6} + C$

$$f'(x) = \sqrt{x} \sin x$$

45. The first derivative of the function f is given above. If $f(0) = 0$, at what value of x does the function f attain its minimum value on the closed interval $[0, 10]$?

(A) 0
(B) 3.14
(C) 4.82
(D) 6.28
(E) 9.42

46. The function f is differentiable with $f(1) = 20$ and $f(4) = -4$. Which of the following statements must be true?

I. $f'(x) < 0$ for all x in the open interval $1 < x < 4$.
II. There exists a number c, where $1 < c < 4$, such that $f(c) = 0$.
III. There exists a number c, where $1 < c < 4$, such that $f'(c) = -8$.

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

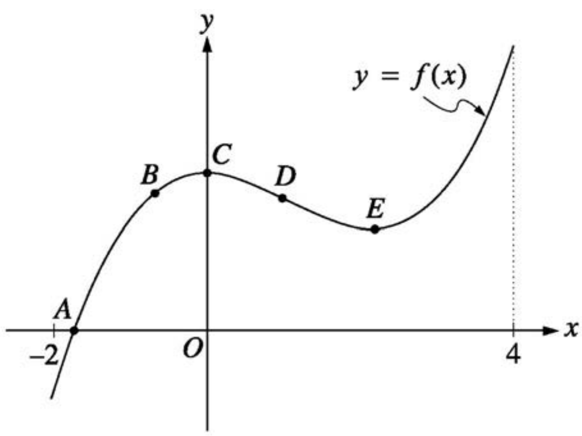
$$g'(x) = \tan\left(\frac{2}{1+x^2}\right)$$

47. Let g be the function with first derivative given above and $g(1) = 5$. If f is the function defined by $f(x) = \ln(g(x))$, what is the value of $f'(1)$?

(A) 0.311
(B) 0.443
(C) 0.642
(D) 0.968
(E) 3.210

48. Let $r(t)$ be a differentiable function that is positive and increasing. The rate of increase of r^3 is equal to 12 times the rate of increase of r when $r(t) =$

(A) $\sqrt[3]{4}$
(B) 2
(C) $\sqrt[3]{12}$
(D) $2\sqrt{3}$
(E) 6



49. The function f is shown in the figure above. At which of the following points could the derivative of f be equal to the average rate of change of f over the closed interval $[-2, 4]$?

(A) A (B) B (C) C (D) D (E) E

50. $\frac{d}{dx}\left(\int_0^{x^2} e^t \, dt\right) =$

(A) e^x

(B) e^{x^2}

(C) $2xe^{x^2}$

(D) $e^{x^2} - 1$

(E) $2xe^{x^2} - 1$

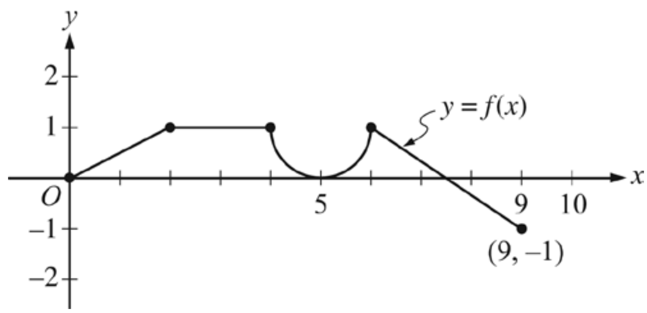
51. A college is planning to construct a new parking lot. The parking lot must be rectangular and enclose 6,000 square meters of land. A fence will surround the parking lot, and another fence parallel to one of the sides will divide the parking lot into two sections. What are the dimensions, in meters, of the rectangular lot that will use the least amount of fencing?

(A) 1,000 by 1,500
(B) $20\sqrt{5}$ by $60\sqrt{5}$
(C) $20\sqrt{10}$ by $30\sqrt{10}$
(D) $20\sqrt{15}$ by $20\sqrt{15}$
(E) $20\sqrt{15}$ by $40\sqrt{15}$

x	0	2	4	6
$f(x)$	9	5	3	1

52. The function f is continuous on the closed interval $[0, 6]$ and has values as shown in the table above. Let L represent the left Riemann sum approximation of $\int_0^6 f(x)\,dx$ with 3 subintervals of equal length, and let R represent the right Riemann sum approximation of $\int_0^6 f(x)\,dx$ with 3 subintervals of equal length. What is $|L - R|$?

(A) 0 (B) 6 (C) 8 (D) 16 (E) 26



53. The graph of the continuous function f consists of three line segments and a semicircle centered at point $(5, 1)$, as shown above. If $F(x)$ is an antiderivative of $f(x)$ such that $F(0) = 2$, what is the value of $F(9)$?

(A) $3+\frac{\pi}{2}$

(B) $9+\frac{\pi}{2}$

(C) $5-\frac{\pi}{2}$

(D) $7-\frac{\pi}{2}$

(E) $\frac{13}{2}-\frac{\pi}{2}$

54. A spherical balloon is being inflated at a constant rate of 25 cm^3/sec. At what rate, in cm/sec, is the radius of the balloon changing when the radius is 2 cm? (The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) $\frac{25}{16\pi}$

(B) $\frac{25}{8\pi}$

(C) $\frac{75}{16\pi}$

(D) $\frac{32\pi}{25}$

(E) $\frac{32\pi}{3}$

55. R is the region below the curve $y = x$ and above the x-axis from $x = 0$ to $x = b$, where b is a positive constant. S is the region below the curve $y = \cos x$ and above the x-axis from $x = 0$ to $x = b$. For what value of b is the area of R equal to the area of S?

(A) 0.739

(B) 0.877

(C) 0.986

(D) 1.404

(E) 4.712

56. Let f be the function defined by $f(x) = e^{3x}$, and let g be the function defined by $g(x) = x^3$. At what value of x do the graphs of f and g have parallel tangent lines?

(A) –0.657

(B) –0.526

(C) –0.484

(D) –0.344

(E) –0.261

57. $\frac{d}{dx}\left(\sin^{-1}(5x)\right) =$

(A) $\cos^{-1}(5x)$

(B) $5\cos^{-1}(5x)$

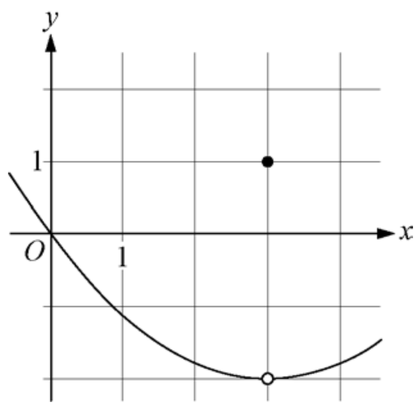
(C) $\frac{1}{\sqrt{1-5x^2}}$

(D) $\frac{5}{\sqrt{1-25x^2}}$

(E) $\frac{5}{1+25x^2}$

58. The population P of bacteria in an experiment grows according to the equation $\frac{dP}{dt} = kP,$ where k is a constant and t is measured in hours. If the population of bacteria doubles every 24 hours, what is the value of k?

(A) 0.029
(B) 0.279
(C) 0.693
(D) 2.485
(E) 3.178



59. The graph of the function f is shown in the figure above. What is the value of $\lim_{x \to 3}(4 - 2f(x))$?

(A) -4
(B) -2
(C) 0
(D) 2
(E) 8

60. During a snowstorm, the rate, in inches per hour, at which snow falls on a certain town is modeled by the function $R(t) = -\cos(t) - 0.9t + 3.0,$ where t is measured in hours and $0 \le t \le 4.$ Based on the model, what is the total amount of snow, in inches, that fell on the town from $t = 0$ to $t = 4$?

(A) 1.2
(B) 1.9
(C) 4.0
(D) 5.6
(E) 18.4

Study Resources

To prepare for the Calculus exam, you should study the contents of at least one introductory college-level calculus textbook, which you can find in most college bookstores. You would do well to consult several textbooks, because the approaches to certain topics may vary. When selecting a textbook, check the table of contents against the knowledge and skills required for this exam.

Visit clep.collegeboard.org/test-preparation for additional calculus resources. You can also find suggestions for exam preparation in Chapter IV of the *Official Study Guide*. In addition, many college faculty post their course materials on their schools' websites.

Answer Key

Section 1		Section 2	
1.	C	36.	B
2.	B	37.	E
3.	D	38.	C
4.	C	39.	B
5.	B	40.	B
6.	D	41.	14
7.	A	42.	B
8.	D	43.	E
9.	D	44.	A
10.	B	45.	D
11.	B	46.	D
12.	B	47.	A
13.	C	48.	B
14.	D	49.	B
15.	A	50.	C
16.	C	51.	C
17.	D	52.	D
18.	D	53.	D
19.	B	54.	A
20.	D	55.	D
21.	E	56.	C
22.	D	57.	D
23.	B	58.	A
24.	B	59.	E
25.	3.05	60.	D
26.	4		
27.	C		
28.	B		
29.	A		
30.	A		
31.	C		
32.	D		
33.	−4		
34.	B		
35.	See below		

35.

x	f is not continuous at x.	f is continuous but not differentiable at x.	f is differentiable at x.
0	✓		
1		✓	
2			✓
3		✓	

Test Measurement Overview

Format

There are multiple forms of the computer-based test, each containing a predetermined set of scored questions. The examinations are not adaptive. There may be some overlap between different forms of a test: any of the forms may have a few questions, many questions, or no questions in common. Some overlap may be necessary for statistical reasons.

In the computer-based test, not all questions contribute to the candidate's score. Some of the questions presented to the candidate are being pretested for use in future editions of the tests and will not count toward his or her score.

Scoring Information

CLEP examinations are scored without a penalty for incorrect guessing. The candidate's raw score is simply the number of questions answered correctly. However, this raw score is not reported; the raw scores are translated into a scaled score by a process that adjusts for differences in the difficulty of the questions on the various forms of the test.

Scaled Scores

The scaled scores are reported on a scale of 20–80. Because the different forms of the tests are not always exactly equal in difficulty, raw-to-scale conversions may in some cases differ from form to form. The easier a form is judged to be, the higher the raw score required to attain a given scaled score. **Table 1** indicates the relationship between number correct (raw score) and scaled score across all forms.

The Recommended Credit-Granting Score

Table 1 also indicates the recommended credit-granting score, which represents the performance of students earning a grade of C in the corresponding course. The recommended B-level score represents B-level performance in equivalent course work. These scores were established as the result of a Standard Setting Study, the most recent having been conducted in 2008. The recommended credit-granting scores are based upon the judgments of a panel of experts currently teaching equivalent courses at various colleges and universities. These experts evaluate each question in order to determine the raw scores that would correspond to B and C levels of performance. Their judgments are then reviewed by a test development committee, which, in consultation with test content and psychometric specialists, makes a final determination. The standard-setting study is described more fully in the earlier section entitled "CLEP Credit Granting" on page 5.

Panel members participating in the most recent study were:

Judy Broadwin	Baruch College of CUNY
Don Campbell	Middle Tennessee State University
Ben Cornelius	Oregon Institute of Technology
John Emert	Ball State University
Daria Filippova	Bowling Green State University
Laura Geary	South Dakota School of Mines and Technology
John Gimbel	University of Alaska — Fairbanks
Stephen Greenfield	Rutgers, The State University of New Jersey
Murli Gupta	George Washington University
Erick Hofacker	University of Wisconsin — River Falls
John Jensen	Rio Salado College
Ben Klein	Davidson College
Stephen Kokoska	Bloomsburg University
Keith Leatham	Brigham Young University
Glenn Miller	Borough of Manhattan Community College
Steven Olson	Northeastern University
David Platt	Front Range Community College
Lola Swint	North Central Missouri College
Mary Wagner-Krankel	St. Mary's University
Rich West	Francis Marion University

After the recommended credit-granting scores are determined, a statistical procedure called scaling is applied to establish the exact correspondences between raw and scaled scores. Note that a scaled score of 50 is assigned to the raw score that corresponds to the recommended credit-granting score for C-level performance, and a high but usually less-than-perfect raw score is selected and assigned a scaled score of 80.

Table 1: Calculus Interpretive Score Data

American Council on Education (ACE) Recommended Number of Semester Hours of Credit: 4

Course Grade	Scaled Score	Number Correct
	80	39-40
	79	38
	78	-
	77	37
	76	36
	75	35-36
	74	35
	73	34
	72	33-34
	71	33
	70	32
	69	31
	68	30-31
	67	30
	66	29
	65	28-29
B	**64**	**27-28**
	63	27
	62	26-27
	61	25-26
	60	24-25
	59	24
	58	23-24
	57	22-23
	56	22
	55	21-22
	54	20-21
	53	19-20
	52	-
	51	18-19
C	**50***	**17-18**
	49	16-17
	48	-
	47	15-16
	46	14-15
	45	13-15
	44	13-14
	43	12-13
	42	11-13
	41	11-12
	40	10-11
	39	9-10
	38	8-10
	37	8-9
	36	7-8
	35	6-8
	34	5-7
	33	5-6
	32	4-6
	31	3-5
	30	2-4
	29	2-3
	28	1-3
	27	0-2
	26	0-1
	25	-
	24	-
	23	-
	22	-
	21	-
	20	-

*Credit-granting score recommended by ACE.

Note: The number-correct scores for each scaled score on different forms may vary depending on form difficulty.

Validity

Validity is a characteristic of a particular use of the test scores of a group of examinees. If the scores are used to make inferences about the examinees' knowledge of a particular subject, the validity of the scores for that purpose is the extent to which those inferences can be trusted to be accurate.

One type of evidence for the validity of test scores is called content-related evidence of validity. It is usually based upon the judgments of a set of experts who evaluate the extent to which the content of the test is appropriate for the inferences to be made about the examinees' knowledge. The committee that developed the CLEP Calculus examination selected the content of the test to reflect the content of Calculus courses at most colleges, as determined by a curriculum survey. Since colleges differ somewhat in the content of the courses they offer, faculty members should, and are urged to, review the content outline and the sample questions to ensure that the test covers core content appropriate to the courses at their college.

Another type of evidence for test-score validity is called criterion-related evidence of validity. It consists of statistical evidence that examinees who score high on the test also do well on other measures of the knowledge or skills the test is being used to measure. Criterion-related evidence for the validity of CLEP scores can be obtained by studies comparing students' CLEP scores with the grades they received in corresponding classes, or other measures of achievement or ability. CLEP and the College Board conduct these studies, called Admitted Class Evaluation Service or ACES, for individual colleges that meet certain criteria at the college's request. Please contact CLEP for more information.

Reliability

The reliability of the test scores of a group of examinees is commonly described by two statistics: the reliability coefficient and the standard error of measurement (SEM). The reliability coefficient is the correlation between the scores those examinees get (or would get) on two independent replications of the measurement process. The reliability coefficient is intended to indicate the stability/consistency of the candidates' test scores, and is often expressed as a number ranging from .00 to 1.00. A value of .00 indicates total lack of stability, while a value of 1.00 indicates perfect stability. The reliability coefficient can be interpreted as the correlation between the scores examinees would earn on two forms of the test that had no questions in common.

Statisticians use an internal-consistency measure to calculate the reliability coefficients for the CLEP exam.[1] This involves looking at the statistical relationships among responses to individual multiple-choice questions to estimate the reliability of the total test score. The SEM is an estimate of the amount by which a typical test-taker's score differs from the average of the scores that a test-taker would have gotten on all possible editions of the test. It is expressed in score units of the test. Intervals extending one standard error above and below the true score for a test-taker will include 68 percent of that test-taker's obtained scores. Similarly, intervals extending two standard errors above and below the true score will include 95 percent of the test-taker's obtained scores. The standard error of measurement is inversely related to the reliability coefficient. If the reliability of the test were 1.00 (if it perfectly measured the candidate's knowledge), the standard error of measurement would be zero.

An additional index of reliability is the conditional standard of error of measurement (CSEM). Since different editions of this exam contain different questions, a test-taker's score would not be exactly the same on all possible editions of the exam. The CSEM indicates how much those scores would vary. It is the typical distance of those scores (all for the same test-taker) from their average. A test-taker's CSEM on a test cannot be computed, but by using the data from many test-takers, it can be estimated. The CSEM estimate reported here is for a test-taker whose average score, over all possible forms of the exam, would be equal to the recommended C-level credit-granting score.

Scores on the CLEP examination in Calculus are estimated to have a reliability coefficient of 0.90. The standard error of measurement is 3.80 scaled-score points. The conditional standard error of measurement at the recommended C-level credit-granting score is 4.18 scaled-score points.

[1] The formula used is known as Kuder-Richardson 20, or KR-20, which is equivalent to a more general formula called coefficient alpha.

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